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Metastable gauged O'Raifeartaigh

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Abstract: We study the possibility of obtaining metastable supersymmetry breaking vacua in a perturbative gauge theory without singlet fields, thus allowing for scenarios where a grand unified symmetry and supersymmetry are broken by the same sector. We show some explicit SU(5) examples. The minimal renormalizable example requires the use of two adjoints, but it is shown to inevitably lead to unwanted light states. We suggest various alternatives, and show that the viable possibilities consist of allowing for non-renormalizable operators, of employing four adjoints or of adding at least one field in a different representation.

Keywords: Spontaneous Symmetry Breaking; Supersymmetry Breaking; GUT.

JHEP04(2008)062

Contents

1. Introduction

The idea of trying to combine grand unified theories with supersymmetry breaking has been used already in the early days of supersymmetry [1 − 4] following mainly the suggestion of dimensional transmutation [5]. The tree order supersymmetry breaking vacuum enforced by the O'Raifeartaigh type superpotential automatically has a flat direction, which gets however stabilized at one loop exactly because of supersymmetry breaking corrections. All of these models are in a perturbative regime and make use of gauge singlets. The mediation of supersymmetry breaking to the MSSM sector is dominated by gravity, which cannot predict (although it can fit) the strong suppression of the flavour changing neutral currents. Later models [6 − 10] were able to get rid of gauge singlets, using nonperturbative gauge sectors to dynamically break supersymmetry. The minima here are not global, but local and thus metastable, although with a long enough lifetime. A typical model has more sectors and gauge groups than usually assumed in phenomenological motivated models like MSSM or grand unification. The results are important and promising: the models considered are mainly realistic and quite natural (without fine-tunings), while the mediation is gauge dominated [1, 3, 11 − 16], an important result.

What we want to explore in this work is the possibility to use as much as possible minimal gauge groups, no singlets and perturbative physics only. The best possibility (and the original motivation) is to use a grand unified group G (we will limit ourselves to SU(5)) without singlets and break both G and $N = 1$ supersymmetry spontaneously (an example of models which break $N = 2$ supersymmetry spontaneously without the use of chiral singlets is given in [18, 19]). At first glance this seems to be in contradiction with what we

JHEP04(2008)062

know from perturbative spontaneous supersymmetry breaking. In fact, one needs a linear term in the superpotential, which must be a singlet, thus naively forbidding for it the use of a gauge multiplet. However, by choosing properly the basis it is easy to get rid of the linear term and thus have a form of the superpotential that can be directly employed in gauge theories without any need for singlets. This will be explicitly shown in section 2. As it will be clear, such a construction is possible only because the considered vacuum is metastable (a recent revival of models with such vacua has been triggered by [17]). For such reasons we will call these models of the metastable gauged O'Raifeartaigh type. It is thus tempting to use this idea in realistic models like for example grand unified theories.

Writing a superpotential that exhibits perturbative and spontaneous supersymmetry breaking without linear terms is only the first part of the story. The second part is to make these metastable gauged O'Raifeartaigh models realistic in the context of grand unified theories. The minimal SU(5) model will be explicitly presented in section 3, together with the main virtues and drawbacks. The virtues are the fact that two adjoint fields suffice to break both supersymmetry and SU(5) gauge symmetry spontaneously. We will show that the model is locally stable in some range of the vevs. One of the vevs is undetermined at tree order, and we will check that it can exhibit a metastable local minimum at one loop. The renormalizable superpotential has two terms only, a form which is enforced by a global $U(1)_R$ symmetry. The drawback of this simple example is the presence of light states, which makes it unrealistic. Possible corrections of this minimal scenario and the role of supergravity will be described in section 4. We will present explicitly three realistic cases in which these unwanted light states are not present: 1) the nonrenormalizable model with two **24**, section 4.1, eq. (4.1); 2) the renormalizable model with four **24**, section 4.2, eq. (4.12); 3) the renormalizable model with two **24** and one **75**, section 4.3, eq. (4.14). Finally, some general remarks and a list of open problems (among which the suggestion to use this type of models in hybrid inflation without singlets) to be discussed in more detail elsewhere will be given in section 5.

2. From singlets to gauge multiplets

We start with the simplest model which exhibits metastable supersymmetry breaking following the general analysis [20]

$$W = S\left(\xi + \lambda\tilde{\phi}^2\right) \ . \tag{2.1}$$

It exhibits a tree level local minimum at

$$\langle\tilde{\phi}\rangle = 0 \quad , \quad S \text{ undetermined} \ , \tag{2.2}$$

providing

$$|\langle S\rangle| \geq \left|\frac{\xi}{2\lambda}\right|^{1/2} \ . \tag{2.3}$$

Such a superpotential cannot be directly written in terms of gauge multiplets, due to the existence of the linear term in S. It is however simple to get rid of it by redefining

$$\tilde{\phi} = \phi - \langle\phi\rangle \ , \tag{2.4}$$

and choosing $\langle\phi\rangle$ such that

$$\xi + \lambda\langle\phi\rangle^2 = 0\ . \tag{2.5}$$

We end up with

$$W = \mu\phi S + \lambda\phi^2 S\ , \tag{2.6}$$

i.e., no linear terms, and with a local minimum at

$$\langle\phi\rangle = -\frac{\mu}{2\lambda} \quad , \quad S \text{ undetermined}\ , \tag{2.7}$$

provided it is in the allowed range

$$|\langle S\rangle| \geq \frac{|\langle\phi\rangle|}{\sqrt{2}}\ . \tag{2.8}$$

This shows that one could start with eq. (2.6), and since there are no linear terms in it, no singlet is really needed: both S and ϕ in (2.6) can be part of a gauge multiplet of a gauge group G, which vevs $\langle S\rangle$ and $\langle\phi\rangle$ break G spontaneously to a subgroup H. In the next section we will give an SU(5) example with two adjoints, both breaking to SU(3)×SU(2)×U(1).

3. The simplest example: two SU(5) adjoints

Using the results in the previous section, we can immediately write down a candidate for a metastable gauged O'Raifeartaigh SU(5) model:

$$W = \mu Tr\Sigma_1\Sigma_2 + \lambda Tr\Sigma_1^2\Sigma_2\ . \tag{3.1}$$

We expand the adjoints Σ_i as

$$\Sigma_i = \begin{pmatrix} O_i + 2\sigma_i/\sqrt{30} & X_i \\ \bar{X}_i & T_i - 3\sigma_i/\sqrt{30} \end{pmatrix}\ , \tag{3.2}$$

where σ_i are the Standard Model (SM) singlets, O_i the color octets $(8,1;0)$, T_i the weak triplets $(1,3;0)$, and $X_i, \bar{X}_i$the color triplet, weak doublets $(3,2;\pm 5/3)$. The vev $v_1 = \langle\sigma_1\rangle$ is obtained from

$$\left\langle \frac{\partial W}{\partial\sigma_1} \right\rangle = 0 \ \rightarrow\ v_1 = \frac{\sqrt{30}}{2}\frac{\mu}{\lambda}\ , \tag{3.3}$$

while supersymmetry breaking is signaled by a nonzero F term:

$$F_2^* \equiv \left\langle \frac{\partial W}{\partial\sigma_2} \right\rangle = \frac{\lambda v_1^2}{\sqrt{30}}\ . \tag{3.4}$$

The other vev, v_2 $(= \langle\sigma_2\rangle)$, is undetermined at tree order, i.e. it is a flat direction. It will be stabilized by nontrivial 1-loop corrections to the Kähler potential, which at tree order is

$$K_0 = Tr\Sigma_i^\dagger\Sigma_i\ . \tag{3.5}$$

Since the vevs of the adjoints are diagonal, the D-terms are vanishing.

JHEP04(2008)062

We have to check two things.

First, that the above model does not contain tachyons. That the singlet has non-negative mass square at least for some choices of the vevs is expected from (2.8). What remains to be checked are the masses of all other SM multiplets. One pair of the bosons in $X_i, \bar{X}_i$ will provide the would-be Nambu-Goldstone bosons (mainly from Σ_2), while the other pair (mainly from Σ_1) will acquire a mass proportional to v_2, so we do not need to worry about them.

After SU(5) breaking, the singlet in Σ_1 gets a supersymmetric mass

$$\mathcal{M}_{\sigma_1} = \frac{1}{\sqrt{30}} 2\lambda v_2 \tag{3.6}$$

while the non-singlet mass matrices have in general the form

$$\mathcal{M} = \frac{\lambda}{\sqrt{30}} \begin{pmatrix} c_2 v_2 & c_1 v_1 \\ c_1 v_1 & 0 \end{pmatrix} \tag{3.7}$$

with $c_1 = 9, c_2 = 6$ for color octets, and $c_1 = 6, c_2 = 9$ for weak triplets. The supersymmetry breaking mass terms in the Lagrangian are

$$\delta L = \frac{\lambda F_2}{\sqrt{30}} \left(-\sigma_1^2 + 2O_1^2 - 3T_1^2 - X_1 \bar{X}_1 \right) + h.c. \tag{3.8}$$

One can now easily find out that there are no tachyonic states if the SM singlet scalar σ_1 is not tachyonic, which is true provided the analogue of (2.8) is satisfied:

$$\sqrt{2}\,|v_2| \geq |v_1| \tag{3.9}$$

The second thing we need to check is whether the flat direction σ_2 gets stabilized at 1-loop following the lines of [5]. All is needed is to check what happens with the wavefunction of the field that breaks supersymmetry (σ_2) [7]. In fact the potential at one loop gets corrected with respect to the tree order one by exactly the wavefunction renormalization (neglecting small finite corrections) through

$$V(\sigma_2) \approx \frac{|F_2|^2}{Z_2(|\sigma_2|)} , \tag{3.10}$$

where F_2 can be read from (3.4) and Z_2 is the wavefunction renormalization at one loop. Obviously the minimum of the potential comes from the maximum of Z_2. At this point one can use the usual rules to write down the renormalization group equations - RGE's (a useful and concise set of rules can be found for example in [15]). For the particle spectrum we take on top of the two adjoints just the minimal set of three generations of matter fields and one pair of 5_H, $\overline{5}_H$ (the results can be easily generalized for more Higgs and/or messenger fields). We obtain $\left(\tau \equiv \frac{1}{8\pi^2} \ln\left(\frac{\mu}{M_{\rm GUT}}\right)\right)$ the following system

$$\frac{d}{d\tau} g_5^{-2} = -2 , \tag{3.11}$$

$$\frac{d}{d\tau} \ln \lambda^2 = -30 g_5^2 + 21\lambda^2 , \tag{3.12}$$

$$\frac{d}{d\tau} \ln Z_2 = 10 g_5^2 - \frac{21}{5}\lambda^2 . \tag{3.13}$$

We have assumed that the couplings between the fundamental and adjoint Higgses are negligible[1]

The extremum of Z_2 fixes one parameter of the superpotential at the minimum

$$\lambda^2 = \frac{50}{21} g_5^2 \; . \tag{3.14}$$

That the extremum of the potential is indeed a minimum can be seen from the negativity of the second derivative at the extremum

$$\frac{1}{Z_2} \frac{d^2 Z_2}{d\tau^2} = -180 g_5^4 \; . \tag{3.15}$$

The minimum (and thus the GUT scale v_2) is determined by the equivalence (3.14).

We have thus checked that the Higgs sector (3.1) can indeed break both SU(5) to the SM gauge group and supersymmetry. Also, the original parameters of the model (μ, λ) can be changed for the physical ones (F, $M_{\rm GUT}$). Notice that all this has been achieved without any fine tuning of the model parameters. The gauge coupling was crucial in this game: the limit of gauge singlets would confirm the observation of [20] that metastable supersymmetry breaking vacua exist only when all values of the flat directions are allowed at tree order. In fact, for $g_5 \to 0$ the one-loop correction would first push v_2 towards the origin, violating the bound (3.9) and eventually finishing in one of the two supersymmetry preserving vacua $v_1 = 0$ or $v_1 = \sqrt{30}\mu/\lambda$ (both with $v_2 = 0$).

The superpotential (3.1) is the most general renormalizable superpotential for two SU(5) adjoints that satisfies a global $\mathrm{U}(1)_R$ symmetry, under which Σ_1 is neutral and Σ_2 has charge 2. This symmetry is spontaneously broken by the v_2 vev and has thus at the perturbative level an exact Nambu-Goldstone boson (σ_2). The R-symmetry must be eventually explicitly broken by supergravity corrections that cancel the cosmological constant [21], which will give a nonzero mass also to this pseudo-Nambu-Goldstone boson.

To summarize: SU(5) is broken at v_2, supersymmetry at v_1. The adjoint Σ_1 could in principle be used as a messenger.

The model is simple and predictive, indeed too predictive, leading to inescapable problems. The most pressing one is that either the supersymmetry breaking scale is comparable to the GUT scale or there are light weak triplets and colour octets mainly from Σ_2. In fact from (3.7) we can see that triplets and octets can have order $M_{\rm GUT}$ mass only if $v_1 = \mathcal{O}(v_2)$, i.e. when $\sqrt{F} \approx v_1 \approx v_2 \approx M_{\rm GUT}$. Since the most obvious candidate for the messengers are the MSSM multiplets in Σ_1, the typical soft mass is only loop (i.e. $\approx 10^{-2}$) suppressed with respect to the triplet and octet masses $\approx F/M_{\rm GUT}$. Keeping v_1 as a free parameter one is still able to unify the gauge couplings, but at a too high scale slightly above 10^{19} GeV, with the sfermion and gaugino masses around 10^5 GeV. Even if one accepted such a high scale, the calculation itself would turn out to be inconsistent, because $M_{\rm GUT} \gtrsim 10^{19}$ GeV would make supergravity corrections to the soft masses dominant. Taking this into account consistently changes very little, making such a model unappealing. In the next section, we describe more realistic scenarios.

[1] This assumption is consistent for example in the simplest of all cases, i.e. $W = \bar{5}_H (y\Sigma_1 + M) 5_H$.

4. More realistic options

We see that the problem arises because the same scale that determines the light SM multiplets (v_1) specifies also the supersymmetry breaking $F \propto v_1^2$ and thus cannot be at the same time large and small. In order to provide for a different scale, one can resort basically to two possibilities: adding non-renormalizable interactions while keeping the field content minimal, or adding more fields and keep renormalizability. We will find three different realistic models, described in sections 4.1, 4.2 and 4.3 respectively. All three of them possess a global $\mathrm{U}(1)_R$ symmetry, broken by the vacuum expectation value of the GUT field that gets a nonzero F term. This is in accordance with the general theorem [22].

4.1 Adding non-renormalizable operators

The first option is to keep the number of adjoints at a minimum but increase the number of interaction terms, i.e. allow for non-renormalizable operators. Using higher powers in Σ_1 is still consistent with the $\mathrm{U}(1)_R$ symmetry. The simplest correction

$$W = Tr\left[\Sigma_2\left(\mu\Sigma_1 + \lambda\Sigma_1^2 + \frac{\alpha_1}{M}\Sigma_1^3 + \frac{\alpha_2}{M}Tr\left(\Sigma_1^2\right)\Sigma_1\right)\right] \tag{4.1}$$

is already enough: one can have large enough vev $v_1 \approx v_2$ but with F arbitrarily low (with a proper fine-tuning of the model parameters), as we now show.

From the equation of motion for σ_1, i.e. $\partial W/\partial\sigma_1 = 0$ we get

$$\mu = \frac{2\lambda}{\sqrt{30}} - \frac{3}{M}\left(\frac{7}{30}\, alpha_1 + \alpha_2\right) v_1^2\ , \tag{4.2}$$

while the second equation $F^* = \partial W/\partial\sigma_2 = 0$ gives

$$F^* = v_1^2\left[\frac{\lambda}{\sqrt{30}} - \frac{2}{M}\left(\frac{7}{30}\alpha_1 + \alpha_2\right) v_1\right]\ . \tag{4.3}$$

This solution has no tachyonic states provided

$$2\left|\frac{v_2}{v_1}\right|^2 \left|\frac{F^*}{v_1^2} - \left(\frac{7}{30}\alpha_1 + \alpha_2\right)\frac{v_1}{M}\right| \geq \left|\frac{F^*}{v_1^2}\right|\ . \tag{4.4}$$

For small enough F this is always the case, so we do not need to worry anymore, allowing large values of v_1. Assuming all parameters real for simplicity we get for the determinants of the octet and triplet mass matrices

$$(\det O)^{1/2} = v_1\left[6\frac{F}{v_1^2} + \left(\frac{75}{30}\alpha_1 + 10\alpha_2\right)\frac{v_1}{M}\right]\ , \tag{4.5}$$

$$(\det T)^{1/2} = v_1\left[4\frac{F}{v_1^2} + \left(\frac{50}{30}\alpha_1 + 10\alpha_2\right)\frac{v_1}{M}\right]\ . \tag{4.6}$$

They depend very mildly on the supersymmetry breaking order parameter F. In the limit $F \to 0$ one has for $v_1 \approx v_2 \approx M_{\mathrm{GUT}}$ the two eigenvalues of the order of M_{GUT}^2/M (barring accidental cancellations). So, although there are intermediate states, they are much less harmful than the ones in the previous examples.

There are various comments in order.

First, notice that in this example there is a fine-tuning needed to split the scales v_1 (that we want to be large in order to avoid too light states) and $\sqrt{F}$ (that we want to be small enough, possibly even around 100 TeV). This is seen for example from the constraint (4.3).

Another important point is that the cutoff M cannot be too large for two reasons: first, eq. (4.3) tells us that λ can be order 1 as required by (3.14) only for mild hierarchies $M_{\rm GUT}/M$; second, one does not want too light intermediate states of mass $M^2_{\rm GUT}/M$.

Finally, one could worry that the new operators introduced could influence the RGE's used to get the minimum of the effective potential. This is not the case, since at one loop the $1/M$ suppressed operators do not contribute to the renormalization of the wave-functions.

Let us now check the gauge coupling unification constraints. The spectrum is the following: at $\Lambda_{\rm SUSY}$ we have the MSSM superpartners (and the second Higgs), at $M^2_{\rm GUT}/M$ we have two colour octets, two weak triplets and a pair of X, $\bar{X}$, i.e. one colour octet, one weak triplet and a full SU(5) adjoint. This is completely analogous to the case described in [23] with the result that the final GUT scale is increased with respect to the usual MSSM case only by a factor 2, if we assume that the cutoff M is 10 times the GUT scale. Due to the increase of $M_{\rm GUT}$ and the appearence of an extra adjoint multiplet at the intermediate scale, the unification gauge coupling α_U increases by about 10% with respect to the usual MSSM case.

4.2 Adding more adjoints

If one wishes to stick to renormalizable models, the simplest idea is to generalize the model (3.1) to something like

$$W = Tr\left[\Sigma_{N+1}\left(\mu_i\Sigma_i + \lambda_{ij}\Sigma_i\Sigma_j\right)\right] \ , \tag{4.7}$$

where now i goes from 1 to some integer N. Notice that λ_{ij} in general does not need to be symmetric, so in general a SU(5) invariant unitary rotation cannot diagonalize λ. For our purpose it is however enough to concentrate just on the diagonal elements of Σ_{N+1} and Σ_i, so that these matrices commute and only the symmetric combination $\lambda_{ij} + \lambda_{ji}$ enters, which can be diagonalized. So we obtain in complete generality the N replica of (3.1), i.e.

$$W = Tr\left[\Sigma_{N+1}\left(\mu_i\Sigma_i + \lambda_i\Sigma_i^2\right)\right] \ . \tag{4.8}$$

Repeating the exercise in section 3, we get

$$v_i = \frac{\sqrt{30}}{2}\frac{\mu_i}{\lambda_i} \ ; \ F_{N+1} = \sum_{i=1}^{N}\frac{\lambda_i}{\sqrt{30}}v_i^2 \ . \tag{4.9}$$

In principle it could be possible to have large v_i but small F_{N+1} (by appropriate fine-tuning of the terms in the sum), but this cannot help, as we shall now see. The mass matrices that generalize (3.7) are now $(N+1)\times(N+1)$ dimensional, and for the triplet and octet have the form

$$\mathcal{M} = \frac{\lambda_i}{\sqrt{30}}\begin{pmatrix} c_{2,i}\delta_{ij}v_2 & c_{1,i}v_i \\ c_{1,i}v_i & 0 \end{pmatrix} \ , \tag{4.10}$$

while the determinant is

$$det\,(\mathcal{M}) = \lambda_1 \ldots \lambda_N \times \sum_{i=1}^{N} c_{2,i} c_{1,i}^2 \lambda_i v_i^2 \; . \tag{4.11}$$

Since all the fields are adjoints, the Clebsch-Gordon coefficients $c_{1,i}, c_{2,i}$ are the same for each SM state, and therefore the sum above is proportional to F_{N+1}: in the limit $F_{N+1} \ll v_{N+1}^2$ we get N masses of order v_{N+1} and one of order F_{N+1}/v_{N+1}. Adding more adjoints in this way cannot give mass to the light colour octets and weak triplets.

This result is a consequence of the superpotential chosen, but there is at least another possibility. Namely, since any nonrenormalizable Lagrangian can be in principle obtained from a renormalizable one by integrating out heavy degrees of freedom, one could use directly the renormalizable potential that gives (4.1). It turns out that, due to the linearity in Σ_2, not one but two additional adjoints (Ω_i) are needed. The following ansatz

$$\begin{aligned} W &= -M Tr\,(\Omega_1 \Omega_2) + Tr\,[\Omega_1\,(\mu_2 \Sigma_2 + \lambda_2 \Sigma_2 \Sigma_1)] \\ &\quad + Tr\,\left[\Omega_2\,\left(\mu_1 \Sigma_1 + \lambda_1 \Sigma_1^2\right)\right] \end{aligned} \tag{4.12}$$

will do the job. One can show that this model has the right properties also in its renormalizable version (without integrating out $\Omega_{1,2}$) for all the mass terms and couplings of order 1. Notice that there is still a $U(1)_R$ symmetry, under which Σ_1 and Ω_1 have charge 0 and Σ_2 and Ω_2 have charge 2. The model could presumably be generalized to

$$W = Tr\,[\Sigma_2\, f_\Sigma(\Sigma_1, \Omega_1)] + Tr\,[\Omega_2\, f_\Omega(\Sigma_1, \Omega_1)] \; . \tag{4.13}$$

We will not push this model any further.

4.3 Adding different representations

There is a further possibility to maintain renormalizability. The point is that what precludes to have really different mass matrices of the MSSM adjoints and the singlet is the absence of enough terms in the superpotential. In other words, there is only one type of trilinear invariants for the adjoint fields (although for three different adjoints there are actually two such invariants, they are equivalent for diagonal elements that commute). So one can try to use different SU(5) representations, and the smallest one for this purpose to add to two adjoints is the **75**. One can write the most general renormalizable superpotential as

$$\begin{aligned} W &= \mu Tr(\Sigma_1 \Sigma_2) + \lambda_1 Tr(\Sigma_1^2 \Sigma_2) \\ &\quad + \lambda_2 Tr(\Phi^2 \Sigma_2) + \eta Tr(\Phi \Sigma_1 \Sigma_2)\,, \end{aligned} \tag{4.14}$$

where Φ is the **75**. The supersymmetry breaking is achieved for the SM singlet vevs

$$\langle \Phi \rangle = -\frac{5\sqrt{15}\eta}{16\lambda_2} v_1\,, \tag{4.15}$$

$$\langle \Sigma_1 \rangle \equiv v_1 = \sqrt{\frac{15}{2}} \frac{\lambda_2 \xi}{\lambda_1 \lambda_2 - \frac{125}{64}\eta^2}\,. \tag{4.16}$$

We get now

$$F_2^* = \frac{1}{\sqrt{30}\lambda_2}\left(\lambda_1\lambda_2 - \frac{125}{64}\eta^2\right) v_1^2 \,, \tag{4.17}$$

which can be fine-tuned to any desired value by fixing the expression in brackets. All one has to do now is to make sure that there are no light states, with masses proportional to the supersymmetry breaking parameter F_2.

There are seven different states in all. Three of these are only present in **75**, namely the $(8, 3; 0)$, the $(3, 1; \pm 10/3)$ and the $(6, 2; \pm 5/3)$. It is evident from the superpotential that they get masses proportional to $\lambda_2 v_2$ since they do not mix. The $X, \bar{X}$ provide the Nambu-Goldstone bosons as before. For the other two, namely the color octets and weak triplets, the determinant of the supersymmetric mass matrices are

$$\det O = -\frac{\sqrt{5}v_2 v_1^2}{\sqrt{6}\lambda_2} \tag{4.18}$$
$$\times\left(\frac{16225}{18432}\eta^4 - \frac{101\sqrt{5}}{12\sqrt{6}}\eta^2\lambda_2\frac{F_2}{v_1^2} + \frac{84}{5}\lambda_2^2\frac{F_2{}^2}{v_1^4}\right)$$

$$\det T = v_1^2\left(\frac{15\sqrt{30}}{32}\frac{\eta^2}{\lambda_2} - 4\frac{F_2}{v_1^2}\right)^2 \tag{4.19}$$

As can be seen, there are no light states left. Thus, this can be considered the minimal renormalizable version.

4.4 Supergravity corrections

In supergravity it is possible to spontaneously break supersymmetry and SU(5) with just one adjoint [24], although with considerable fine-tuning. In this paper we want to take the opposite limit, i.e. to avoid the domination of terms suppressed by the Planck mass. However, supergravity is there, if nothing else, to cancel the cosmological constant. Here we will shortly check what supergravity does to our models. We will limit ourselves to the most delicate aspects of the above scenario, i.e. the stability of the minimum found through the RGE's and to the R-axion mass.

Consider the nonrenormalizable model with two adjoints. Although the model has a cutoff lower than the Planck scale, we assume that the UV completion at this cutoff, valid all the way to M_{Pl}, maintains at least approximately the form of the SM singlets' superpotential

$$W = F(\phi_i)\sigma_2 + W_0(\phi_i) \,, \tag{4.20}$$

where $F(\langle\phi_i\rangle) \equiv F$ sets the scale of supersymmetry breaking and $W_0(\langle\phi_i\rangle) \equiv W_0 \approx FM_{Pl}$ fine-tunes the cosmological constant to zero. Assuming that all vevs are smaller than M_{Pl} the typical supergravity contribution to the potential for σ_2 is schematically $F^2(\sigma_2/M_{Pl})^n$ and so only the lowest n's are relevant. The correction to the mass is

$$\Delta m_{\sigma_2}^2 \approx \frac{4}{3}\left(\frac{F}{M_{Pl}}\right)^2 \,, \tag{4.21}$$

to be compared with the mass found in the global supersymmetric case. This can be easily read off from (3.10) and (3.15)

$$m_{\sigma_2}^2 \approx 360 \left(\frac{\alpha_U}{4\pi}\right)^2 \left(\frac{F}{M_{\rm GUT}}\right)^2 . \tag{4.22}$$

We see that the mass square from the solution (4.22) in the global supersymmetry case is numerically (for $M_{\rm GUT} \approx 4.10^{16}$ GeV, $M_{Pl} \approx 2.10^{18}$ GeV, $\alpha_U \approx 1/20$) 15 times or so bigger than the supergravity contribution (4.21). So, the mass is stable.

There is however a new linear contribution and this represents the main danger. The main part of the potential can be written schematically as the sum of the leading gauge contribution and the supergravity corrections

$$V \approx m_{\sigma_2}^2 (\sigma_2 - M_{\rm GUT}^0)^2 + \frac{F^2}{M_{Pl}} \sigma_2 + \dots \tag{4.23}$$

In the limit $M_{Pl} \to \infty$ we had $M_{\rm GUT}^0 = \langle \sigma_2 \rangle \equiv M_{\rm GUT}$, but now the true minimum gets shifted as (we omit numbers of order one)

$$M_{\rm GUT} = M_{\rm GUT}^0 + \frac{F^2}{m_{\sigma_2}^2 M_{Pl}} . \tag{4.24}$$

The two contributions are of the same order and the supergravity one could even dominate. To settle it one would need to perform a more precise calculation. One can however notice that the value $M_{\rm GUT}^0$ was defined as the scale, at which the equality (3.14) is satisfied. But then it is enough to shift this scale to a different value, so that the final $M_{\rm GUT}$ (4.24) is what we would like it to be.

Another issue is the R-axion mass. The constraint of a vanishing cosmological constant requires a constant term of order FM_{Pl} in the superpotential. This term explicitly breaks the U(1) R-symmetry. The pseudo R-axion gets thus a non-vanishing mass of order [21]

$$m_a^2 \approx \frac{F^2}{M_{\rm GUT} M_{Pl}} . \tag{4.25}$$

Whether the model is cosmologically safe or not depends on the value of F. A weak scale R-axion mass is dangerous, for similar reasons as moduli, see for example [25 − 27] for possible solutions in this case. In the opposite case of small F the R-axion mass is harmless.

A short comment is due on D-terms. It is known, that in general $N = 1$ $D = 4$ supergravity, the D and F terms are connected [28, 29]. This means, that if D-terms are non-zero, they are related to F-terms. In our case the adjoints are diagonal, so their D-terms are still zero, similar to the global limit.

5. Conclusions

We have shown that it is possible to construct realistic superpotentials that break perturbatively both supersymmetry and a gauge symmetry without using singlets. This is

JHEP04(2008)062

possible only because the minima considered were metastable. For such models there is no reason to introduce extra gauge sectors, which dynamically break supersymmetry. One can thus study just simple gauge groups, a particularly appealing situation in case of grand unified theories. The price to pay is that extra states need to be introduced.

We found three different realistic SU(5) examples:

1. nonrenormalizable model with two **24**, section 4.1, eq. (4.1);
2. renormalizable model with four **24**, section 4.2, eq. (4.12);
3. renormalizable model with two **24** and one **75**, section 4.3, eq. (4.14).

All three of them have a spontaneously broken $U(1)_R$ global symmetry.

There are many issues not touched in this paper, to be addressed in subsequent work. Let us mention some of them.

The doublet-triplet problem. In the minimal case of the renormalizable model with two adjoints the doublets and triplets of a single pair of 5_H and $\bar{5}_H$ cannot be split enough even with fine tuning. In fact, Σ_2 should not couple to the fundamentals, because its F term destabilizes the weak scale. On the other side Σ_1 has a too small vev (of the order of $\sqrt{F}$) to split enough the doublets and triplets. It is thus reassuring that in the realistic versions this problem disappears, since now v_1 can be of the order of the GUT scale.

In models with 75 one can use the missing partner mechanism [30]. Such a model has quite some number of huge representations (two 24_H, one 75_H and one pair of 50_H and $\overline{50}_H$), but it should be stressed that no fine-tuning is needed, except the obvious one that creates the hierarchy $\sqrt{F} \ll M_{\rm GUT}$, needed in all known perturbative supersymmetry breaking models without light states.

Mediation of supersymmetry breaking. The obvious mediators in all these type of models are the heavy gauge bosons and the adjoints. They can dominate over gravity only for relatively low $M_{\rm GUT}$, not much higher than the usual in MSSM. The large number of fields can help for this purpose. Notice that the potential problem of negative soft mass squared is not necessarily there due to the subsequent running, as shown recently in [31]. Other possible contributions need the introduction of extra (possibly intermediate scale) states, like the usual extra pairs of SU(5) fundamental and anti-fundamentals, or a pair of 15_H and $\overline{15}_H$ that can be used also for the neutrino masses [32].

Non-perturbative contributions. We have assumed that the perturbative part of the superpotential dominates. One could ask, how can the non-perturbative contributions influence the picture. Can one calculate them? The models considered are realistic and thus necessarily complicated enough to make the usual techniques (use of holomorphicity, symmetries, etc) hard and probably non conclusive. Notice that none of the models we presented is ultraviolet free. The best one can do without further work is to make the most sensitive part of our mechanism, i.e. the presence of a $U(1)_R$ symmetry, independent on the quantum non-perturbative corrections. This can be guaranteed by making the $U(1)_R$ global symmetry non-anomalous. Of course this depends on the model chosen. For example, in the non-renormalizable model with two adjoints, one needs to add to the usual spectrum (two adjoint Higgses, a pair of fundamental Higgses and three generations of 10_F and $\bar{5}_F$ matter) also two pairs of $(5_i+\bar{5}_i)$ chiral multiplets with vanishing R-charge (enforced for example by a term $\lambda_i \bar{5}_i \Sigma_2 5_i$ in the superpotential). A general treatment of this issue is very interesting, but beyond the scope of this paper.

Vacuum metastability. We have assumed throughout the paper that the vacuum lifetime is longer than the age of the universe. This can be checked either with an explicit calculation using the full 1-loop effective potential, or estimated as it is done in [7]. Using the constraint (4.4) and the method in [7] one finds for such a bounce action an approximate value of $S_B \approx 2\pi^2 M_{\rm GUT}^2/|F|$, which is much larger than the required value of ≈ 500 needed for the lifetime to be longer than the age of the universe.

Different gauge groups. We have limited ourselves to the prototype example of a SU(5) grand unified theory. For many aspects the SO(10) GUT is more successful. Unfortunately the minimal renormalizable version [33] cannot break at the same time the gauge group and supersymmetry, the reason being the absence of a flat direction. An extra problem in such nonminimal groups is the need for breaking rank, which typically needs an extra fine-tuning.

A special role can be played here by partial unified groups, like the Pati-Salam or the Left-Right group. Being possible at lower scales without being necessarily worried about proton decay constraints, they can automatically give a low enough supersymmetry breaking scale without any fine-tuning. The minimal model with two fields can work in both cases, however, again an additional sector (and additional fine-tuning) is needed in order to break rank. Of course the whole motivation for supersymmetry is here less pronounced: no hierarchy problem because of little or no hierarchy, no one-step unification because of intermediate scales.

Inflation without singlets. It is interesting that these type of models give possible candidates for a non-singlet (although still MSSM singlet) inflaton. Apart from few exceptions (for example [34] in MSSM and [35] in a GUT) this would be one of the very few examples of such inflatons on the market. The simplest model (3.1) is very similar to the prototype model of F-term hybrid inflation [36]. If one is not too ambitious and does not pretend that the same model describes also supersymmetry breaking, this simple model could in principle work. In fact, in order to get rid of the unwanted light states, one can think that the final state after inflation is in the true minimum, in which both adjoints become heavy. Preliminary results seem to confirm that inflation can indeed take place, in a similar manner as in the case with singlets introduced in [36]. For example, one can calculate the derivatives of the 1-loop potential and find out that the usual requirements for inflation to happen are satisfied. What would be particularly interesting is to see if there are any differences in predictions with respect to the case with a singlet. This work is in progress and a detailed analysis will be presented elsewhere.

JHEP04(2008)062

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